Math 1920 **Test on Chapter 7** **Name______Solutions__________**

Each problem has a value of 5 points. Show your work in the space provided … continue your work on the back side of the question sheets if necessary. Use of calculators and other devices is not allowed.

1. Evaluate $\int_0^{\pi/2} \cos(x)\sin(2x)\,dx$ if possible. [Hint: sin(2x) = 2sin(x) cos(x)]

$$\int_0^{\pi/2} \cos(x)\sin(2x)\,dx = -2\int_0^{\pi/2} \cos^2 x(-\sin x\,dx) = \left.\frac{-2\cos^3 x}{3}\right|_0^{\pi/2} = \frac{2}{3}$$

2. Evaluate $\int_0^{\infty} \frac{1}{(x+1)^3}\,dx$ if possible.

$$\int_0^{\infty} \frac{1}{(x+1)^3}\,dx = \lim_{t\to\infty}\int_0^{t} \frac{1}{(x+1)^3}\,dx = \lim_{t\to\infty}\left.\frac{-1}{2(x+1)^2}\right|_0^{t} = \frac{1}{2}$$

3. Evaluate $\int_0^{1} \frac{4x^2+1}{(x+1)^3}\,dx$ if possible.

$$\frac{4x^2+1}{(x+1)^3} = \frac{4}{x+1} - \frac{8}{(x+1)^2} + \frac{5}{(x+1)^3}$$

$$\int_0^{1} \frac{4x^2+1}{(x+1)^3}\,dx = \left.\left(4\ln(x+1) + \frac{8}{x+1} - \frac{5}{2(x+1)^2}\right)\right|_0^{1} = 4\ln(2) - \frac{17}{8}$$

Alternative approach: make the substitution u=x+1

4. Evaluate $\int_0^{1} \frac{y}{e^{2y}}\,dx$ if possible. [Hint: how would you integrate xe^x?]

$\int_0^{1} \frac{y}{e^{2y}}\,dx = \int_0^{1} y e^{-2y}\,dx$ by parts with $u = y$ and $dv = e^{-2y}\,dy$

$$\int_0^{1} y e^{-2y}\,dx = \left.\frac{-ye^{-2y}}{2}\right|_0^{1} + \frac{1}{2}\int_0^{1} e^{-2y}\,dy = \frac{1}{4} - \frac{3}{4e^2}$$

5. Evaluate $\int_0^2 \frac{1}{x-1}dx$ if possible.

$$\int_0^2 \frac{1}{x-1}dx = \int_0^1 \frac{1}{x-1}dx + \int_1^2 \frac{1}{x-1}dx$$

$$\int_0^1 \frac{1}{x-1}dx = \lim_{t\to 1^-} \int_0^t \frac{1}{x-1}dx = \lim_{t\to 1^-} \ln|x-1||_0^t = \lim_{t\to 1^-} \ln|t-1| = \infty$$

so the improper integral diverges

6. Evaluate $\int \tan^3(x)dx$ if possible. [Hint: $1+\tan^2(x)=\sec^2(x)$]

$$\int \tan^3(x)dx = \int (sec^2 x-1)tanx\,dx = \int (sec^2 x)tanx\,dx - \int \tan x\,dx = \frac{\tan^2 x}{2} + \ln|\cos x| + C$$

7. Evaluate $\int x^3\sqrt{1-x^2}dx$ if possible.

Let $x=\sin(\theta)$. Then $dx=\cos(\theta)d\theta$

$$\int x^3\sqrt{1-x^2}dx = \int \sin^3\theta\cos^2\theta\,d\theta = \int \sin\theta\left(1-\cos^2\theta\right)\cos^2\theta\,d\theta = \frac{-\cos^3(\theta)}{3} + \frac{\cos^5(\theta)}{5} + C$$

unwinding the substitution, $\cos(\theta)=\sqrt{1-x^2}$ so

$$\int x^3\sqrt{1-x^2}dx = \frac{(1-x^2)^{\frac{3}{2}}}{3} - \frac{(1-x^2)^{\frac{5}{2}}}{5} + C$$

8. Evaluate $\int \csc^4(x)\cot^3(x)dx$ if possible.

$$\int \csc^4 x\cot^3 x\,dx = \int \left(1+\cot^2 x\right)\cot^3 x\,csc^2 x\,dx = -\int \left(\cot^3 x + \cot^5 x\right)d(\cot x)$$

$$= -\frac{\cot^4 x}{4} - \frac{\cot^6 x}{6} + C$$

or

$$\int \csc^4 x\cot^3 x\,dx = \int \left(\frac{1}{\sin^4 x}\right)\frac{\cos^3 x}{\sin^3 x}dx = \int \frac{1-\sin^2 x}{\sin^7 x}\cos x\,dx$$

$$= \frac{\sin^{-6} x}{-6} - \frac{\sin^{-4} x}{-4} + C$$

9. Evaluate $\int \ln(x^2)\,dx$ if possible.

Use integration by parts with $u=\ln(x^2)$ $dv=dx$

$$\int \ln(x^2)\,dx = x\ln x^2 - \int 2\,dx = x\ln x^2 - 2\mathrm{x} + C$$

If you wish, you can simplify first using the fact that $\ln(x^2)=2\ln x$

10. Evaluate $\int \frac{1}{1+e^x}\,dx$ by first making the substitution u=e^x to express the integrand as a rational function. Then find the antiderivative of this rational function and reverse the substitution to express your answer in terms of the original variable x. [Recall: if u=e^x then x=ln(u).]

$$\int \frac{1}{1+e^x}\,dx = \int \frac{1}{1+u}\frac{1}{u}\,du \quad \text{since} \quad dx = \frac{1}{u}\,du$$

$$\int \frac{1}{1+u}\frac{1}{u}\,du = \int \frac{-1}{1+u} + \frac{1}{u}\,du = -\ln|1+u| + 1\ln|u| + C = -\ln(1+e^x) + x + C$$